C19 sol

First compute the characteristic polynomial,

$$\begin{aligned}\mathcal{P}\mathcal{C}(\boldsymbol{x}) &= \det(\mathcal{C} - \boldsymbol{x}\boldsymbol{I}_2)\\ &= \begin{vmatrix} -1-x & 2 \\ -6 & 6-x \end{vmatrix}\\ &= (-1-x)(6-x) - (2)(-6)\\ &= x^2 - 5x + 6\\ &= (x-3)(x-2)\end{aligned}$$

So the eigenvalues of C are the solutions to $pc(x) = 0$, namely, $\lambda = 2$ and $\lambda = 3$.To obtain the eigenspaces, construct the appropriate singular matrices and find expressions for the null spaces of these matrices.

$$\lambda = 2$$

$$C - (2)I_2 = \begin{pmatrix} -3 & 2 \\ -6 & 4 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & -\frac{2}{3} \\ 0 & 0 \end{pmatrix}$$

$$\mathcal{E}\mathcal{C}(2) = \mathcal{N}(\mathcal{C} - (2)I_2) = \left\langle \left\{ \begin{pmatrix} \frac{2}{3} \\ 1 \end{pmatrix} \right\} \right\rangle = \left\langle \left\{ \begin{pmatrix} \frac{2}{3} \\ 2 \end{pmatrix} \right\} \right\rangle$$

$$\lambda = 3$$

$$C - (3)I_2 = \begin{pmatrix} -4 & 2 \\ -6 & 3 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & -\frac{1}{2} \\ 0 & 0 \end{pmatrix}$$

$$\mathcal{E}\mathcal{C}(3) = \mathcal{N}(\mathcal{C} - (3)I_2) = \left\langle \left\{ \begin{pmatrix} \frac{1}{2} \\ 1 \end{pmatrix} \right\} \right\rangle = \left\langle \left\{ \begin{pmatrix} 1 \\ 2 \end{pmatrix} \right\} \right\rangle$$

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C19 sol

Primero compute la característica polinomial

$$\begin{aligned}\mathcal{P}\mathcal{C}(\boldsymbol{x}) &= \det(\mathcal{C} - \boldsymbol{x}\boldsymbol{I}_2)\\ &= \begin{vmatrix} -1-x & 2 \\ -6 & 6-x \end{vmatrix}\\ &= (-1-x)(6-x) - (2)(-6)\\ &= x^2 - 5x + 6\\ &= (x-3)(x-2)\end{aligned}$$

De esta forma los valores propios de C son la solución a $pc(x) = 0$, llamado, $\lambda = 2$ y $\lambda = 3$.Para obtener los espacios propios, construya la matriz singular apropiada y encuentre expresiones para los espacios inutiles de estas matrices.

$$\lambda = 2$$

$$C-(2)I_2 = \begin{pmatrix} -3 & 2 \\ -6 & 4 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & -\frac{2}{3} \\ 0 & 0 \end{pmatrix}$$

$$\mathcal{E}\mathcal{C}\,(2) = \mathcal{N}(\mathcal{C}-(2)I_2) = \left\langle \left\{ \begin{pmatrix} \frac{2}{3} \\ 1 \end{pmatrix} \right\} \right\rangle = \left\langle \left\{ \begin{pmatrix} \frac{2}{3} \\ 2 \end{pmatrix} \right\} \right\rangle$$

$$\lambda = 3$$

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